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OPTICAL CONSTANTS OF GERMANIUM



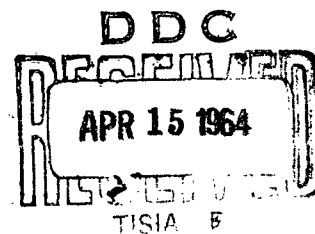
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NOTICES

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OPTICAL CONSTANTS OF GERMANIUM

ABSTRACT

An analysis has been made of the effects of change of refractive index of germanium on radiometric calibration; this analysis shows that the index cannot change by more than 0.01 if 2% accuracy is to be maintained—unless calibration procedures are used to account for this change. Several methods of measuring index have been investigated; as a necessary preliminary, the effects of changes in temperature and pressure on index were evaluated. For the usual atmospheric conditions, variations of 50°C or 50 torr will cause changes of no more than ± 0.00001 of the refractive index of air. If temperature or pressure measurements can have errors as large as 50°C or 50 torr, then the error or uncertainty in index can be as large as 10^{-5} . Similarly, the sample must be measured to $\pm 0.04^\circ\text{C}$.

Each of nine possible techniques has been analyzed in terms of the precision necessary in the measurement of each of the independent variables. Deviation techniques are discussed in Section 3.1, reflection techniques in Section 3.2, interferometric methods in Section 3.3, and a sphere method in Section 3.4. The required measurement accuracies are summarized in Table I. Transmission measurements made on a sample submitted by the sponsor are also described and analyzed.

1 INTRODUCTION

The NIMBUS satellite, being developed by NASA and the Weather Bureau to determine basic and dynamic properties of our atmosphere and the development of weather conditions, includes in its instrumentation a grating spectrometer and infrared radiometers which use germanium-immersed thermistor bolometers. Both types of instrument are meant to measure with a radiometric accuracy of 2% or better. The NIMBUS satellite will undergo the usual temperature variations of orbiting vehicles: a variation of about -60°C to $+60^\circ\text{C}$. Accordingly, it is of interest to know how the refractive index of germanium and the absorption vary with temperature.

The approach to solving this problem has been to investigate the many different methods of measuring refractive index to determine which will provide the most reliable, accurate, and economical answers. A comprehensive literature search on the properties of germanium was also undertaken, and some measurements were made of the transmission of poor germanium samples. This report is divided into three main categories: the evaluation of different methods of measuring the refractive index of germanium, transmission measurements, and the bibliography.

REFRACTIVE INDEX MEASUREMENTS

This section omits a discussion of the more usual simple techniques: Pulfrich, Abbé, optical path measurement, and critical angle measurement. The more precise techniques have been considered in greater detail; these include the following: variation deviation techniques, interferometric means, and reflective measurements like those of Robinson and others. The analysis is based on a desire to measure index to 0.00001; the results, however, are in a more general form: one that gives the required precision in a measured value for any accuracy.

Although the extremely accurate measurement of refractive index is an important academic problem, it is also useful to investigate just how precise the index measurement must be to obtain a 2% radiometric accuracy. This analysis is also included below.

2.1. METHOD OF ERROR ANALYSIS

The refractive index is a function of several variables:

$$n = n(x, y, \dots).$$

If one of these variables changes by an unknown amount, it can be written as its original value plus the change. Thus the index is written as

$$n + \Delta n = n(x + \Delta x, y + \Delta y, \dots).$$

The change in index, Δn , is written as

$$\Delta n = n(x + \Delta x, y + \Delta y, \dots) - n(x, y, \dots).$$

Clearly, this is exactly the expression for the differential of a function of several variables before the limits are taken. This is the basis for using differential calculus in error analysis. However, the negligibility of second-order difference terms must be verified by one means or another. For example, the change in total radiant emittance of a body should be written

$$\Delta W = \sigma(\epsilon + \Delta\epsilon) (T + \Delta T)^4 - \sigma T^4$$

When expanded, this is written

$$\Delta W = \sigma(\epsilon + \Delta\epsilon)[4T^3 \Delta T + 6T^2(\Delta T)^2 + 4T(\Delta T)^3 + (\Delta T)^4] + \sigma T^4 \Delta\epsilon$$

The terms which include ΔT to a power greater than one and terms like $\Delta\epsilon \Delta T$ must be small compared to ΔT and $\Delta\epsilon$ terms. Then the expression obtained by differentiation is valid:

$$dW = \sigma T^3 \Delta T + \sigma T^4 \Delta\epsilon$$

If the second-order differences are small, calculus is valid and the chain rule applies:

$$dn = \frac{\partial n}{\partial x} dx + \frac{\partial n}{\partial y} dy + \dots$$

The relative error can be written $\frac{dn}{n}$.

2.2. EFFECT OF INDEX CHANGE ON RADIOMETRIC ACCURACY

In order to analyze the effect on radiometric accuracy of a change in refractive index, one can start with the basic formula for irradiance (assuming no absorption). Radiance in an optical system is constant if there are no losses. Assuming that the only loss is at the front surface of the immersion lens, the irradiance of the detector is given by $H = N(1 - R)\omega$. The change is obtained by differentiation:

$$dH = N[(1 - R)d\omega - \omega dR]$$

$$\frac{dH}{H} = \frac{d\omega}{\omega} - \frac{dR}{1 - R}$$

Thus there are two influences: change of reflection loss, and change of solid angle. The reflectivity error of an uncoated sample is found by assuming that the radiation is incident normally. Thus the equations for reflectivity and its errors are

$$R = \left(\frac{n - 1}{n + 1}\right)^2 \quad 1 - R = \frac{2n}{(n + 1)^2}$$

$$dR = 2\left(\frac{n - 1}{n + 1}\right) \left[\frac{(n + 1) - (n - 1)}{(n + 1)^2} \right] dn = 4 dn \frac{n - 1}{(n + 1)^3}$$

$$\frac{dR}{1 - R} = \frac{4 dn(n - 1)(n + 1)^2}{(n + 1)^3 2n} = 2 dn \frac{(n - 1)}{n + 1}$$

The error in solid angle due to a change in critical angle is obtained by considering the change in critical angle as a function of temperature. The critical angle α is obtained from Snell's Law by assuming that the refracted angle is $\pi/2$:

$$\sin \alpha = n'/n$$

Here n' is the index of selenium and n is that of germanium. The error obtained by differentiation is found as follows:

$$\cos \alpha d\alpha = \frac{ndn' - n'dn}{n^2}$$

$$d\alpha = \left(\frac{dn'}{n} - \frac{n'}{n^2} dn \right) \frac{1}{\cos \alpha}$$

The solid angle is proportional to the square of the critical angle, so that one has the following relation:

$$\omega = k\alpha^2$$

$$d\omega = 2k\alpha d\alpha$$

The relative error is found by division:

$$\frac{d\omega}{\omega} = \frac{2 d\alpha}{\alpha}$$

By combining the above equations, the relative error can then be computed to be

$$\frac{d\omega}{\omega} = \frac{2 \left(\frac{dn'}{n} - \frac{n' dn}{n^2} \right)}{\cos \alpha \arcsin \frac{n'}{n}} = \frac{2 \frac{dn'}{n'} - \frac{dn}{n}}{\sqrt{1 - (n/n')^2} \arcsin \frac{n'}{n}}$$

Note that $\cos \alpha$ is equal to $\sqrt{1 - (n/n')^2}$, so that the relative change in irradiance is written

$$\frac{dH}{H} = \frac{\frac{dn'}{n'} - \frac{dn}{n}}{1/2 \arcsin \frac{n'}{n} \sqrt{(n/n')^2 - 1}} + \frac{2 dn(n-1)}{n+1}$$

Numerical evaluation is now possible. If n is 4 and n' is 2.4 (for germanium and selenium respectively), then

$$\frac{dH}{H} = \frac{0.4 dn' - 0.25 dn}{1/2 \arcsin 0.6 \sqrt{25/9 - 1}} + \frac{2 dn(n-1)}{n+1} = 0.9 dn' + 0.63 dn$$

Therefore, a total change of 0.02 requires knowledge to about 0.01 in the refractive index of both the germanium and the substrate.

2.3. EFFECTS OF TEMPERATURE AND PRESSURE CHANGES ON INDEX CHANGES

One of the basic problems in the analysis of methods for index measurement is an evaluation of the effects of the uncertainty in knowledge of the temperature and pressure of air. The equation relating the index of air to temperature and pressure [1, 2] is given as

$$(n_1 - 1) = (n_{15,760} - 1) \frac{P(1 + \beta_t P)(1 + 15 \alpha)}{760(1 + 760\beta_{15})(1 + \alpha T)}$$

The constants have the following values:

$$\beta_t = (1.05 - 0.015 T) 10^{-6}$$

$$\beta_{15} = 0.813 \times 10^{-6}$$

$$\alpha = 0.0037$$

$n_{15,760} = n'$ is the index of air at 15°C and 760 mm Hg.

Substitution yields

$$n_1 - 1 = 0.001387 (n' - 1) [P + (1.05 - 0.015T) 10^{-6} P^2] (1 + 0.0037T)^{-1}$$

The uncertainty in the index of air with respect to temperature is found by the usual technique of differentiation. The partial derivative with respect to temperature is

$$\frac{\partial n_1}{\partial T} = \frac{(n' - 1)(0.001387) \{ (1 + 0.0037T)(-0.015 \times 10^{-6} P^2) - (0.0037)[(P + 1.05 - 0.015T) 10^{-6} P^2] \}}{(1 + 0.0037T)^2}$$

Assuming as before that T is 373°K, P is 760 mm Hg, and $n' - 1$ is 280×10^{-6} , the resulting value is 1.936×10^{-7} . The variation of index with respect to pressure is found as follows:

$$\frac{\partial n_1}{\partial P} = \frac{(n' - 1)(0.001387)}{1 + 0.0037T} [1 + 2(1.05 - 0.015T) 10^{-6} P]$$

If it is assumed that $T = 373^\circ\text{K}$ and $P = 760$ mm Hg, as before, the numerical result is 1.62×10^{-7} . The total change is given by the total derivative:

$$dn_1 = 1.936 \times 10^{-7} dT + 1.62 \times 10^{-7} dP$$

Pressure and temperature variations of less than about 50°C or 50 mm Hg will cause an error of no more than ± 0.00001 .

A similar analysis can be applied to the sample temperature. Since $dn_2 = n dn_1 + n_1 dn$, where $n = \frac{n_2}{n_1}$, n_1 must be known to 2.5×10^{-6} . Since $\frac{dn_2}{dT}$ is about 2.68×10^{-4} from earlier measurements [3] and since $\frac{dn_1}{dT}$ is around three orders of magnitude smaller than this, then the temperature accuracy needed is determined by the change of index of germanium with temperature:

$$dn_2 = 2.68 \times 10^{-4} dT$$

So n_2 will be accurate to ± 0.00001 only within a temperature range of about 0.037°C . Alternatively,

$$\begin{aligned} dn &= \frac{1}{n_1} \frac{\partial n_2}{\partial T} dT - \frac{n_2}{n_1} \frac{\partial n_1}{\partial T} dT \\ &= 2.68 \times 10^{-4} dT - 4 \times 1.936 \times 10^{-7} dT \\ &\approx 2.68 \times 10^{-4} dT \end{aligned}$$

The uncertainty in temperature can be no greater than 1 part in 27, or 0.037 .

3 DISCUSSION AND EVALUATION OF VARIOUS REFRACTIVE INDEX MEASUREMENT METHODS

3.1. DEVIATION METHODS

Methods which measure the deviation of a beam of light from some known direction are among the oldest and most accurate known. The classical method of minimum deviation, discussed in every good text on optics, is notable for its simplicity and elegance. It is not necessary to obtain minimum deviation for the determination of index; in some applications it is useful not to do so (the technique then may simply be called "prism deviation"). Also, some simplification is achieved by measuring deviations for two prism positions. There is then an almost endless number of possible variations of the techniques; all sorts of geometric figures may be used. Minimum deviation, prism deviation, and two-position deviation will be discussed in this section.

3.1.1. MINIMUM DEVIATION METHOD. By the usual manipulations [4] one obtains the following expression for minimum deviation (Figure 1 defines the symbols):

$$n_2 = \frac{n_1 \sin 1/2(\delta_m + \alpha)}{\sin \alpha/2}$$

The total derivative is given by

$$dn_2 = \frac{\partial n_2}{\partial n_1} dn_1 + \frac{\partial n_2}{\partial \delta} d\delta_m + \frac{\partial n_2}{\partial \alpha} d\alpha$$

The partial derivatives are as follows:

$$\frac{\partial n_2}{\partial n_1} = \frac{\sin 1/2(\delta_m + \alpha)}{2 \sin \alpha/2}$$

$$\frac{\partial n_2}{\partial \delta_m} = \frac{n_1 \cos 1/2(\delta_m + \alpha)}{2 \sin \alpha/2}$$

$$\frac{\partial n_2}{\partial \alpha} = \frac{n_1 \sin \alpha/2 \cos 1/2(\delta_m + \alpha) - \sin 1/2(\delta_m + \alpha) \cos \alpha/2}{\sin^2 \alpha/2}$$

$$\frac{\partial n_2}{\partial \alpha} = \frac{n_1 \sin (\alpha + \delta_m)/2}{\sin^2 \alpha/2}$$

Therefore the total derivative is

$$dn_2 = \frac{\sin 1/2(\delta_m + \alpha)}{\sin \alpha/2} dn_1 + \frac{n_1 \cos 1/2(\delta_m + \alpha)}{2 \sin \alpha/2} d\delta_m + \frac{n_1 \sin \delta_m/2}{\sin^2 \alpha/2} d\alpha$$

At this stage it is necessary to make some estimates of δ_m and α because these enter into each of the partial derivatives. This is done by assuming nominal values (4 and 1, respectively) for the indexes of germanium and air. Then

$$4 = \frac{\sin 1/2(\delta_m + \alpha)}{\sin \alpha/2}$$

Because this equation has no unique solution, some judicious choices must be made. If $\delta_m + \alpha$ is 180° , then α must be about 30° and δ_m is 150° . This is usually too large a dispersion angle for most experimental arrangements; the prism angle would be sufficiently large to make the optical path sufficiently thick, so that there is too much absorption. If the prism angle is 10° , then the minimum deviation angle is 30° ; this is more reasonable. By the original considerations, one has

$$\frac{\partial n_2}{\partial n_1} = \frac{\sin 1/2(\delta_m + \alpha)}{\sin \alpha/2} = 4$$

$$\frac{\partial n_2}{\partial \delta_m} = \frac{n_1 \cos 1/2(\delta_m + \alpha)}{2 \sin \alpha/2} = \frac{1(0.940)}{2(0.087)} \frac{(0.470)}{(0.087)} = 5.402$$

$$\frac{\partial n_2}{\partial \alpha} = \frac{n_1 \sin \delta_m/2}{\sin^2 \alpha/2} = \frac{0.259}{2(0.087)^2} = \frac{0.1295}{(0.087)^2} = \frac{0.1295}{0.007569} = 17.10$$

Since n_1 is not strongly dependent on temperature, the assumptions permit numerical evaluation of the equations written above which yields

$$dn_2 = 4 dn_1 + 5.402 d\delta_m + 17.10 d\alpha + 2.68 \times 10^{-4} dT$$

The angles δ_m and α are expressed here in radians. Therefore δ_m must be measured to 2×10^{-3} mrad and α must be measured to 5×10^{-4} mrad to obtain 1 part in the fifth decimal. In more familiar units, δ_m must be measured to about one half second of arc and α measured to one eighth second of arc.

The question of wavelength uncertainty remains. The dispersion curve for any optical material is nonlinear. (If it were linear, one index measurement at each of two wavelengths would suffice for all spectral measurements and this research problem would be almost trivial.) Therefore, errors in wavelength are proportional to index errors, but the "constant" of proportionality is a function of wavelength. In the region around 2μ (where $dn/d\lambda$ is large), errors in wavelength are serious; at larger wavelengths they are not. By taking published values, one can learn something about the permissible values. From the data [5], values of $\frac{dn}{d\lambda}$ range from 0.0001 at about 15μ to 0.1 at about 2μ . A reasonable midband value of 0.01 yields

$$\frac{\partial n}{\partial \lambda} d\lambda = 0.01 d\lambda$$

Therefore $d\lambda$ must be less than 0.001 in this region; in fact a curve can be constructed. Some of the points are listed in Table II. The worst case requires spectral accuracy to 2.5 \AA , and this is short of 3μ where there are enough emission lines.

3.1.2. PRISM DEVIATION METHOD. In this method a beam of light is refracted through the prism at any deviation—not necessarily minimum deviation. The geometry of the prism provides the first equation:

$$\theta'_1 + \theta'_2 = \alpha$$

Snell's law provides the next pair of equations:

$$n_1 \sin \theta_1 = n_2 \sin \theta'_1$$

$$n_1 \sin \theta_2 = n_2 \sin \theta'_2$$

Now a series of substitutions puts these equations in a form that simplifies solution of n_2/n_1 :

$$\begin{aligned}
n_1 \sin \theta_1 &= n_2 \sin (\alpha - \theta'_2) \\
&= n_2 (\sin \alpha \cos \theta'_2 - \cos \alpha \sin \theta'_2) \\
&= n_2 \left[\sin \alpha (1 - \sin^2 \theta'_2)^{1/2} - \cos \alpha \sin \theta'_2 \right]
\end{aligned}$$

$$\sin \theta'_2 = \frac{n_1}{n_2} \sin \theta_2$$

$$\sin \theta_1 = \sin \alpha \left[\left(\frac{n_2}{n_1} \right)^2 - \sin^2 \theta_2 \right]^{1/2} - \cos \alpha \sin \theta_2$$

$$\left(\frac{n_2}{n_1} \right)^2 = \sin^2 \theta_2 + \frac{\sin^2 \theta_1 + 2 \sin \theta_1 \cos \alpha \sin \theta_2 + \cos^2 \alpha \sin^2 \theta_2}{\sin^2 \alpha}$$

$$\left(\frac{n_2}{n_1} \right)^2 = \frac{\sin^2 \theta_2 + \sin^2 \theta_1 + 2 \sin \theta_1 \cos \alpha \sin \theta_2}{\sin^2 \alpha}$$

$$n_2 = \frac{n_1}{\sin \alpha} \left(\sin^2 \theta_1 + \sin^2 \theta_2 + 2 \sin \theta_1 \sin \theta_2 \cos \alpha \right)^{1/2}$$

This is the "simple" form for n_2 (at least it is an explicit representation of n_2 or n_2/n_1 in terms of prism and refraction angles). The total derivative of n_2 (a function of the angles θ_1 and θ_2), α , and the index n_1 must now be found:

$$dn_2 = \frac{\partial n_2}{\partial n_1} dn_1 + \frac{\partial n_2}{\partial \theta_1} d\theta_1 + \frac{\partial n_2}{\partial \theta_2} d\theta_2 + \frac{\partial n_2}{\partial \alpha} d\alpha$$

The evaluation proceeds as follows:

$$\frac{\partial n_2}{\partial n_1} = \frac{1}{\sin \alpha} \left(\sin^2 \theta_1 + \sin^2 \theta_2 + 2 \sin \theta_1 \sin \theta_2 \cos \alpha \right)^{1/2}$$

$$\frac{\partial n_2}{\partial \theta_1} = \frac{n_1 (\sin \theta_1 \cos \theta_1 + \cos \theta_1 \sin \theta_2 \cos \alpha)}{\sin \alpha \left(\sin^2 \theta_1 + \sin^2 \theta_2 + 2 \sin \theta_1 \sin \theta_2 \cos \alpha \right)^{1/2}}$$

$$\frac{\partial n_2}{\partial \theta_2} = \frac{n_1 (\sin \theta_2 \cos \theta_2 + \sin \theta_1 \cos \theta_2 \cos \alpha)}{\sin \alpha \left(\sin^2 \theta_1 + \sin^2 \theta_2 + 2 \sin \theta_1 \sin \theta_2 \cos \alpha \right)^{1/2}}$$

$$\frac{\partial n_2}{\partial \alpha} = -n_1 \frac{\sin \theta_1 \sin \theta_2 + \cos \alpha (\sin^2 \theta_1 + \sin^2 \theta_2)}{\sin^2 \alpha \left(\sin^2 \theta_1 + \sin^2 \theta_2 + 2 \sin \theta_1 \sin \theta_2 \cos \alpha \right)^{1/2}}$$

Now it is necessary to make some assumptions about the actual experimental arrangements.

Assume that n_1 is 1, n_2 is 4, and θ_1 and θ_2 are 20.4° . Numerical substitution shows that

$$\frac{\partial n_2}{\partial n_1} = 4, \frac{\partial n_2}{\partial \theta_1} = 5.37, \frac{\partial n_2}{\partial \theta_2} = 5.37, \text{ and } \frac{\partial n_2}{\partial \alpha} = 22.7. \text{ Therefore,}$$

$$dn_2 = 4 dn_1 + 5.37 d\theta_1 + 5.37 d\theta_2 + 22.7 d\alpha$$

The results are that

$$\begin{aligned} n_1 &\text{ must be measured to } 2.5 \times 10^{-6}; \\ \theta_1 \text{ and } \theta_2 &\text{ must be measured to } 1.86 \times 10^{-6}; \\ \alpha &\text{ must be measured to } 4.4 \times 10^{-7}. \end{aligned}$$

With the assumptions made above, this method is about equivalent to minimum deviation because θ_1 and θ_2 were assumed equal; this is not always required, however. In fact, the situation is such that as θ_1 gets larger, so does θ_2 , and the error increases. This is not particularly important, since the experiment would be set for minimum deviation, but no care would be taken to determine that such was the case. The error analysis for minimum deviation is then essentially correct.

3.1.3. TWO-POSITION PRISM METHOD. The Servo Corporation of America reports a scheme for measuring refractive index that incorporates a reflecting side on the prism [6]. The arrangement is shown in Figure 2.

The prism angle is α , the rotation angle is ϵ , and their sum is β . The deviation angle is δ ; therefore the angle relationships are:

$$\begin{aligned} \delta &= -\theta_3 + \theta_1 & \beta &= \alpha + \epsilon \\ \theta'_2 &= -\theta_2 & \theta_3 &= \beta \end{aligned}$$

The Snell relations are

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta'_1 \\ n_1 \sin \theta_3 &= n_2 \sin \theta'_3 \end{aligned}$$

Therefore

$$n_1 \sin \beta = n_2 \sin \theta'_3$$

From the trigonometry of the triangles,

$$\theta'_3 + \theta_2 + \pi - \alpha = \pi$$

$$\theta'_3 + \theta_2 = \alpha$$

$$\theta_2 - \theta'_1 = \alpha$$

When these equations are combined, the following useful relation is obtained:

$$\theta'_3 = 2\alpha - \theta'_1$$

Therefore

$$\begin{aligned} n_1 \sin \beta &= n_2 \sin (2\alpha - \theta'_1) \\ &= n_2 \sin \left[2\alpha - \arcsin \left(\frac{n_1 \sin \theta_1}{n_2} \right) \right] \end{aligned}$$

Simplification is accomplished thus:

$$\begin{aligned} \sin \beta &= \frac{n_2}{n_1} (\sin 2\alpha \cos \theta'_1 - \cos 2\alpha \sin \theta'_1) \\ \sin \theta'_1 &= \frac{n_1}{n_2} \sin \theta_1 \\ \cos \theta'_1 &= \frac{n_1}{n_2} \left[\left(\frac{n_2}{n_1} \right)^2 - \sin^2 \theta_1 \right]^{1/2} \\ \sin \beta &= \sin 2\alpha \left[\left(\frac{n_2}{n_1} \right)^2 - \sin^2 \theta_1 \right]^{1/2} - \cos 2\alpha \sin (\beta + \delta) \end{aligned}$$

Solving for n_2/n_1 is accomplished thus:

$$\begin{aligned} \sin^2 \beta + 2 \sin \beta \sin (\beta + \delta) \cos 2\alpha + \cos^2 2\alpha \sin^2 (\beta + \delta) &= \sin^2 2\alpha \left[\left(\frac{n_2}{n_1} \right)^2 - \sin^2 \theta_1 \right] \\ \left(\frac{n_2}{n_1} \right)^2 &= \frac{\sin^2 \beta + 2 \sin \beta \sin (\beta + \delta) \cos 2\alpha + \cos^2 2\alpha \sin^2 (\beta + \delta) + \sin^2 2\alpha \sin^2 (\delta + \beta)}{\sin^2 2\alpha} \\ n^2 &= \frac{\sin^2 \beta + 2 \sin \beta \sin (\beta + \delta) \cos 2\alpha + \sin^2 (\beta + \delta)}{\sin^2 2\alpha} \end{aligned}$$

An error analysis is then made for this technique in the usual way.

$$n = \frac{n_2}{n_1} = \frac{[\sin^2 \beta + 2 \sin \beta \sin(\beta + \delta) \cos 2\alpha + \sin^2(\beta + \delta)]^{1/2}}{\sin^2 \alpha}$$

$$dn = \frac{\partial n}{\partial \beta} d\beta + \frac{\partial n}{\partial \alpha} d\alpha + \frac{\partial n}{\partial \delta} d\delta$$

$$\begin{aligned} \frac{\partial n}{\partial \beta} &= \frac{2 \sin \beta \cos \beta + 2 \cos \beta \sin(\beta + \delta) \cos 2\alpha + 2 \sin \beta \cos(\beta + \delta) \cos 2\alpha + 2 \sin(\beta + \delta) \cos(\beta + \delta)}{2 \sin 2\alpha [\sin^2 \beta + 2 \sin \beta \sin(\beta + \delta) \cos 2\alpha + \sin^2(\beta + \delta)]^{1/2}} \\ &= \frac{\sin \beta \cos \beta + \cos \beta \sin(\beta + \delta) \cos 2\alpha + \sin \beta \cos(\beta + \delta) \cos 2\alpha + \sin(\beta + \delta) \cos(\beta + \delta)}{\sin 2\alpha [\sin^2 \beta + 2 \sin \beta \sin(\beta + \delta) \cos 2\alpha + \sin^2(\beta + \delta)]^{1/2}} \end{aligned}$$

$$\begin{aligned} \frac{\partial n}{\partial \alpha} &= \frac{(\sin 2\alpha)^{1/2} [X]^{-1/2} (-4 \sin \beta \sin(\beta + \delta) \sin 2\alpha) - [X]^{1/2} 2 \cos 2\alpha}{\sin^2 2\alpha} \\ &= \frac{-2 \sin \beta \sin(\beta + \delta) \sin^2 2\alpha - 2 \sin^2 \beta \cos 2\alpha - 4 \sin \beta \sin(\beta + \delta) \cos^2 2\alpha - 2 \sin^2(\beta + \delta) \cos 2\alpha}{\sin^2 2\alpha [\sin^2 \beta + 2 \sin \beta \sin(\beta + \delta) \cos 2\alpha + \sin^2(\beta + \delta)]^{1/2}} \\ &= \frac{-2[\sin \beta \sin(\beta + \delta) \sin^2 2\alpha + \sin^2 \beta \cos 2\alpha + 2 \sin \alpha \sin(\beta + \delta) \cos^2 2\alpha + \sin^2(\beta + \delta) \cos 2\alpha]}{\sin^2 2\alpha [\sin^2 \beta + 2 \sin \beta \sin(\beta + \delta) \cos 2\alpha + \sin^2(\beta + \delta)]^{1/2}} \\ \frac{\partial n}{\partial \delta} &= \frac{\sin \beta \cos(\beta + \delta) \cos 2\alpha + \sin(\beta + \delta) \cos(\beta + \delta)}{\sin 2\alpha [\sin^2 \beta + 2 \sin \beta \sin(\beta + \delta) \cos 2\alpha + \sin^2(\beta + \delta)]^{1/2}} \\ &= \frac{\cos(\beta + \delta) [\sin \beta \cos 2\alpha + \sin(\beta + \delta)]}{\sin 2\alpha [\sin^2 \beta + 2 \sin \beta \sin(\beta + \delta) \cos 2\alpha + \sin^2(\beta + \delta)]^{1/2}} \end{aligned}$$

Numerical evaluation is accomplished by assuming that

$$\alpha = 15^\circ$$

$$\epsilon = 5^\circ$$

$$\delta = 30^\circ$$

$$2\alpha = 30^\circ$$

$$\frac{\partial n}{\partial \beta} = 3.025$$

$$\frac{\partial n}{\partial \alpha} = 7.93$$

$$\frac{\partial n}{\partial \delta} = 1.268$$

Because n_2 is a function of temperature (besides being a function of the geometry), and because n_1 is not strongly temperature- or pressure-dependent, the result is

$$n_2 = nn_1 + f(T)$$

where T is temperature. Then

$$dn_2 = \frac{\partial n_2}{\partial n_1} dn_1 + \frac{\partial n_2}{\partial n} dn + \frac{\partial n_2}{\partial T} dT$$

$$dn_2 = n dn_1 + n_1 dn + \frac{\partial n_2}{\partial T} dT$$

For germanium and air, the index values are $n \approx 4$, $n_1 \approx 1$, and $\frac{\partial n_2}{\partial T} \approx 2.68 \times 10^{-4}$. Therefore

$$dn_2 = 4 dn_1 + 3.025 d\beta + 7.93 d\alpha + 1.268 d\delta + 2.68 \times 10^{-4} dT$$

As before, n_1 must be measured to an accuracy of 2.5×10^{-6} and T must be measured to 0.037°C ; β must be measured to 3.3×10^{-6} rad (or about 0.68 second of arc); α must be measured to 1.26×10^{-6} rad (or about 0.27 second of arc); δ must be measured to 7.89×10^{-6} rad (or about 1.63 seconds of arc).

3.2. REFLECTION METHODS

In the early 1950's it was found that the optical constants of materials with metallic properties could be determined by measuring the reflection coefficient and the phase relationship between the two components of the reflected electric vector—one component parallel and the other perpendicular to the plane of incidence (see Figure 3). There are basically three ways in which data obtained in this way can be used to determine the optical constants: the Ditchburn method, the Tousey method, and the Robinson method. These methods and their limitations are described below.

3.2.1. DITCHBURN [7] METHOD. If the reflecting surface is free of any contamination, one can relate the phase difference, Δ , between the two different orientations of the electric vector and the ratio of their amplitudes R_p/R_n (parallel to perpendicular) to the optical constants of the material. This relationship is

$$\frac{1 - e^{i\Delta} \tan \Psi}{1 + e^{i\Delta} \tan \Psi} = \sqrt{\frac{n^2 - k^2 - \sin^2 \phi - i2nk}{\tan \phi \sin \phi}}$$

where $\tan^2 \Psi = \frac{R_p}{R_n}$, ϕ is the angle of incidence, and the complex index of refraction is $\tilde{n} = n - ik$.

Separating the real and imaginary parts, one obtains

$$n^2 - k^2 = \tan^2 \phi \sin^2 \phi \left[\frac{\cos^2 2\psi - \sin^2 2\psi \sin^2 \Delta}{(1 + \sin 2\psi \cos \Delta)^2} + \sin^2 \phi \right]$$

and

$$2nk = \tan^2 \phi \sin^2 \phi \frac{\sin 4\psi \sin \Delta}{(1 + \sin 2\psi \cos \Delta)^2}$$

Therefore n and k can be calculated from these last two equations, on the basis of experimental values of Δ and ψ .

The phase Δ depends upon ϕ and k ; Δ is small for small ϕ , and gradually increases from 0° to 180° as ϕ changes from 0 to 90° . As k decreases, the region of transition of Δ from low to high values becomes more pronounced; when k is zero, Δ is a step function; i.e., 0° to 180° at the Brewster angle. Thus when k is low, the phase angle is difficult to determine (possibly only to $\pm 0.1^\circ$); the accuracy of the ϕ determination from geometrical considerations is about 1 arcsec. The error analysis is accomplished in the usual way. First, the two equations above are solved for n :

$$n^4 - n^2 \left[\frac{\tan^2 \phi \sin^2 \phi (\cos^2 2\psi - \sin^2 2\psi \sin^2 \Delta)}{(1 + \sin 2\psi \cos \Delta)^2} + \sin^2 \phi \right] = \frac{\tan^4 \phi \sin^4 \phi \sin^2 4\psi \sin^2 \Delta}{4(1 + \sin 2\psi \cos \Delta)^4}$$

The following assumptions are made: $\phi = 60^\circ$, $\psi = 26.5^\circ$, $\Delta = 5^\circ$, $n = 4$.

The value for $\partial n / \partial \phi$ is then found to be 0.1. The choice of parameter values is based upon a reasonable geometric arrangement and the theory presented by Ditchburn. The variation in n due to changes in ψ can be found in the same way: $\partial n / \partial \psi$ is -0.118. Then $\partial n / \partial \Delta$ is found to be 0.0338. Thus

$$dn = 0.1 d\phi + 0.034 d\Delta - 0.118 d\psi$$

If ϕ can be measured to one arcsec, Δ within one arcmin and ψ to 0.02 arcmin, then

$$dn = 0.0011178$$

This implies that, for wavelengths shorter than 1.5μ , the method permits determining the optical constants with an accuracy of 10^{-3} if the effects of surface films such as oxides and surface irregularities are negligible. However, the work of Avery [8] and also that of Archer [9] indicates that the best accuracy obtainable at these shorter wavelengths is about 2%. It is therefore unrealistic to assume that the method will be applicable to the longer wavelengths with sufficient accuracy.

In order for dn to be $\sim 10^{-5}$, the accuracy desired, $d\phi$ must be no greater than 10^{-5} degrees, $d\psi$ must be no greater than 10^{-5} degrees, and $d\Delta$ must not exceed 3×10^{-5} degrees. In terms of arcseconds, the accuracy must be at least

$$\begin{cases} d\phi < 0.36 \text{ arcsec} \\ d\psi < 0.36 \text{ arcsec} \\ d\Delta < 1.08 \text{ arcsec} \end{cases}$$

Having to know $d\phi$ this accurately could be quite a problem, since surface irregularities could cause errors that exceed the required precision. Also, the incident beam must be collimated very carefully. Polishing the sample strains the surface; this then necessitates careful annealing techniques. These surface strains have been known to introduce inaccuracies of as much as 5% in refractive index measurements of PbS by reflection techniques. To date, the error in determining ψ (due to unavoidable experimental problems) amounts to about 3.6 arcsec, or an order of magnitude greater than the minimum acceptable value of 0.36 arcsec.

The values given above are for $\lambda < 1.5 \mu$ or, in a region of relatively large extinction coefficient, $k > 0.5$. The extinction coefficient in the region of interest is around 10^{-7} . This renders the determination of n a difficult task at best.

Another problem encountered when using this method to determine refractive index is the effect of film formation on the surface of the sample. Coatings such as oxides and contaminants are commonplace and extremely difficult to avoid. According to Archer [9], both Δ and ψ are affected by the presence of surface films. In his measurements a 10-Å oxide film on Ge caused a 2% to 30% error in k and a 0.6% to 5% error in n . Thin oxide films obviously can greatly disturb the measurements in the second significant figure of the refractive index. This is quite reasonable, since the measured refractive index in this method is simply the index of the sample surface, inasmuch as the reflection mode is being used.

3.2.2. TOUSEY [10] METHOD. This method, too, employs reflection measurements made for two different orientations of the electric vector.

$$\frac{R_p}{R_n} = \frac{a^2 + b^2 - 2a \sin \phi \tan \phi + \sin^2 \phi \tan^2 \phi}{a^2 + b^2 + 2a \sin \phi \tan \phi + \sin^2 \phi \tan^2 \phi}$$

where

$$2a^2 = \{[(n^2 - k^2) - \sin^2 \phi]^2 + 4n^2 k^2\}^{1/2} + [(n^2 - k^2) - \sin^2 \phi]$$

$$2b^2 = \{[(n^2 - k^2) - \sin^2 \phi]^2 + 4n^2 k^2\}^{1/2} - [(n^2 - k^2) - \sin^2 \phi]$$

Now the measurement of $\frac{R_p}{R_n}$ at two angles of incidence will provide two equations with two unknowns:

$$R_1 = f_1(n, k)$$

$$R_2 = f_2(n, k)$$

These equations are solved graphically. Tousey states that the uncertainty in n is about 0.01. This is 10^3 times worse than the maximum allowable uncertainty.

3.2.3. ROBINSON [11] METHOD. Just as the phase and attenuation of the output of an electrical network are interdependent, so is the phase and attenuation of the reflected electric vector. Thus one can use reflection at normal incidence. The Fresnel equation for the amplitude of reflected light at normal incidence from a flat surface is

$$\bar{r} = \frac{(n - ik) - 1}{(n - ik) + 1} = |\bar{r}|e^{i\theta}$$

The reflection coefficient R is

$$R = |\bar{r}|^2$$

If $|\bar{r}|$ is known over the entire wavelength spectrum, then θ at any signal frequency ω_0 can be determined from the Kramers-Kronig relation

$$\theta(\omega_0) = \frac{2\omega_0}{\pi} \int_0^\infty \frac{\ln|\bar{r}(\omega)|}{\omega^2 - \omega_0^2} d\omega$$

The success of the method depends on the fact that negligible error results from a lack of knowledge of all parts of the frequency spectrum. This technique has been used satisfactorily for the wavelength interval where the extinction coefficient is quite high (i.e., on the short wavelength side of the absorption edge). In the region of interest, however, k is 10^{-7} and the imaginary part of the Fresnel coefficients vanishes; the result is that the refractive index measurement reverts to a determination of n by using the Brewster angle θ_B :

$$n = \tan \theta_B$$

Again the problems of surface films and incident angle determinatives are encountered.

3.3. INTERFEROMETRIC METHODS

The essential idea of interferometers involves dividing the light from a source into two or more beams which are then superposed; the irradiance in the region of superposition varies from point to point between maxima (which exceed the sum of the irradiances in the beams), and

minima (which may be zero). Several different instruments are available which will divide a single beam in order to exhibit the interference pattern [12]. The mathematics of the interference phenomena is not dependent on any particular instrument; thus the conditions for fringes to be formed will be discussed in a general way.

The irradiance has been defined as the time average of the amount of energy which crosses, in unit time, a unit area perpendicular to the direction of the energy flow. Then the total irradiance for two beams propagating in the same direction is

$$H = H_1 + H_2 + 2\sqrt{H_1 H_2} \cos \delta$$

where δ is the phase difference between the two waves.

Evidently there will be maxima of irradiance where

$$|\delta| = 0, 2\pi, 4\pi, \dots,$$

and minima of irradiance where

$$|\delta| = \pi, 3\pi, \dots,$$

If a plane parallel plate of transparent material is illuminated by an infinitely distant point source or a collimated beam (see Figure 4), some point P on the same side as S is reached by two rays—one reflected at the upper surface and the other at the lower surface—so that there is an interference pattern. The two rays from S to P travel different optical paths having a difference of ΔL . Then the corresponding difference in phase is

$$\pm\delta = \frac{2\pi}{\lambda} \Delta L \pm \pi$$

where $\pm\pi$ is the phase advance or retardation from reflection. From the geometry in Figure 4,

$$\Delta L = n'(\overline{AB} + \overline{BC}) - n\overline{AD}$$

where n' and n are the refractive indexes of the plate and the surrounding medium. If t is the thickness of the plate, and I, I' are the angles of incidence and refraction, then

$$\overline{AB} = \overline{BC} = t/\cos I'$$

$$\overline{AD} = \overline{AC} \sin I = 2t \tan I' \sin I$$

$$n \sin I = n' \sin I'$$

then

$$\Delta L = 2n't \frac{1 - \sin^2 I'}{\cos I'}$$

$$\Delta L = 2n't \cos I'$$

Therefore

$$\pm \delta = \frac{4\pi}{\lambda} n't \cos I' \pm \pi$$

There are bright fringes when

$$\pm \delta = \pm 2m\pi$$

There are dark fringes when

$$\pm \delta = \pm (2m + 1)\pi$$

This corresponds to bright fringes when

$$2n't \cos I' = \left(m + \frac{1}{2}\right)\lambda$$

There are dark fringes when

$$2n't \cos I' = m\lambda$$

In each case

$$m = 0, 1, 2, \dots$$

A given fringe is characterized by a constant value of I' and therefore a constant value of I because

$$n \sin I = n' \sin I'$$

The condition for circular fringes is

$$\cos I' = \sqrt{1 - \left(\frac{n}{n'}\right)^2 \sin^2 I}$$

3.3.1. SIMPLE INTERFEROMETRIC METHOD. The refractive index is found (on the basis of the theory above) to be

$$n' = \sqrt{\left(\frac{m_0 \lambda}{2t}\right)^2 + n^2 \sin^2 I}$$

where

$$m_0 = m \quad \text{or} \quad m \pm \frac{1}{2}$$

The differential error may be written as

$$dn' = \frac{1}{n'} \left[\left(\frac{d\lambda}{\lambda} + \frac{dm_o}{m_o} - \frac{dt}{t} \right) \left(\frac{m_o \lambda}{2t} \right)^2 + n \sin^2 I \, dn + n^2 \sin I \cos I \, dI \right]$$

In performing the actual experiment, the angle of incidence will be almost zero and the refractive index of air is nearly one. The wavelength will be allowed to vary from 1 to 15 μ . The values chosen are $n' = 4$, $t = 5$ mm, and $I = 4$ arcmin. The allowable errors are then $dI = 3$ arcmin, $dt = 1.25 \text{ } \mu$, $d\lambda = 10^{-5} \mu$, $dm_o = 0.01$, and $dn = 10$.

One way to evaluate the usefulness of this method is to examine the restriction placed on the parameters. The severest restriction placed on this method is the measurement of length, (i.e., parallelism). Obtaining a narrow source is also a problem. The use of a line source (e.g., a hydrogen gas discharge) is ruled out since there are no sources of this type in the spectral region of interest. If there were an ideal source, still another problem would be introduced: determining m ($4 \times 10^4 \leq m \leq 2.5 \times 10^3$), the order of interference. If the desired resolution can be obtained and the number of fringes can be determined, the next step would be to determine the thickness to $\pm 125 \text{ } \text{\AA}$; this is impossible, as was shown in the section on reflection. Also, the two faces of the sample must not make an angle greater than 0.001 second of arc; they must be extremely parallel, because of the restriction on thickness.

The refractive index of air presents no great problem.

The next error is that concerning the angle of incidence, which, like the refractive index of air, does not introduce any great problem. In fact, at small angles the angular resolution required decreases and would be negligible for an angle of (for example) less than 1° off normal. This happens because the sine function decreases at small angles and is the major contributor to the angular error.

3.3.2. ROTATING SAMPLE METHOD. Another experiment could be constructed which employs two plates; one plate rotates with respect to the other. The refractive index of a transparent plate and its dispersion can be determined by the use of white and monochromatic light. The determination of the dispersion is based upon the shift between the true and the apparent position of the center of the system of fringes formed by white light [13].

A plate is cut in half, and one of the halves is rotated slowly about an axis perpendicular to the beam; the fringes begin to move because of the increase in the optical pathlength (see Figure 5). From the geometry, the difference in optical path can be written as

$$\Delta L = 2[n'(\overline{OC} - \overline{OA}) + n(\overline{CE} - \overline{AB})]$$

where n' , n are the refractive indexes of the plate and the surrounding medium. If t is the thickness of the plate and I , I' are the angles of incidence and refraction, then

$$\overline{OC} = t/\cos I', \quad \overline{OA} = t$$

$$\overline{CE} = (\overline{BD} - \overline{CD}) \sin I$$

$$\overline{BD} - \overline{CD} = t(\tan I - \tan I')$$

$$\overline{CE} = t\left(\frac{1}{\cos I'} - \frac{n}{n' \cos I'}\right) \sin^2 I$$

$$\overline{AB} = t/\cos I = t$$

$$\Delta L = 2t[n'(\cos I' - 1) - n(\cos I - 1)]$$

From Snell's law,

$$n \sin I = n' \sin I'$$

Combining these equations gives

$$\sqrt{n'^2 - n^2 \sin^2 I} = n(\cos I - 1) + n' + \frac{m_o \lambda}{2t}$$

Squaring both sides and solving for n' produces

$$n' = \frac{(2n^2 t - nm_o \lambda)(1 - \cos I) + \frac{(m_o \lambda)^2}{4nt}}{2nt(1 - \cos I) - \frac{m_o \lambda}{n}}$$

or

$$n' = n_{\text{air}} + \frac{\frac{m_o \lambda \cos I}{4nt} + \frac{(m_o \lambda)^2}{4nt}}{2t(1 - \cos I) - \frac{m_o \lambda}{n}}$$

where m_o is $m + \frac{1}{2}$, or m and $m = 0, 1, 2, \dots$

The partial derivatives of n' will be useful in evaluating the errors:

$$D^2 \frac{\partial n'}{\partial t} = \frac{(m_o \lambda)^3}{4n^2 t^2} - m_o \lambda \left(\frac{m_o \lambda}{nt} + 2 \cos I \right) (1 - \cos I)$$

$$D^2 \frac{\partial n'}{\partial \lambda} = \left(\frac{m_o^2 \lambda}{n} + 2 m_o t \cos I \right) (-\cos I) - \frac{m_o^3 \lambda^2}{4n^2 t}$$

$$D^2 \frac{\partial n'}{\partial m} = \left(\frac{m_o \lambda^2}{n} + 2 \lambda t \cos I \right) (1 - \cos I) - \frac{m_o^2 \lambda^3}{4n^2 t}$$

$$D^2 \frac{\partial n'}{\partial I} = \left(\frac{m_o \lambda}{2n} - 2t \right) m_o \lambda \sin I$$

$$D^2 \frac{\partial n'}{\partial n} = \left\{ \frac{1}{2} \left(\frac{m_o \lambda}{n} \right)^2 + 4t^2 (1 - \cos I) - 4t \left(\frac{m \lambda}{n} \right) \right\} (1 - \cos I)$$

where

$$D = 2t (1 - \cos I) - \frac{m_o \lambda}{n}$$

These expressions assume that the instrument is in perfect alignment, that the plates are parallel, and that the source is collimated. Obviously these conditions are not attainable in any real experiment. The limitations on the accuracies are discussed in Section 3.3.1.

By setting $n' = 4$, the following results can be obtained: $I \approx 3^\circ$, $m = 10$ at $\lambda = 2 \mu$, and $t = 5$ mm. Then $dI = 0.5$ arcsec, $dt = 100 \text{ \AA}$, $d\lambda = 2 \times 10^{-6} \mu$, $dm_o = 2 \times 10^{-5}$ fringe, and $dn_{\text{air}} = 5 \times 10^{-7}$.

The limitations by this method are the same as in Section 3.3.1 except that the number of fringes to be counted is on the order of 10; the method still does not seem physically realizable.

3.3.3. RELATIVE INDEX MEASUREMENT. Another method can be devised to reduce the number of fringes that must be counted. If the refractive index is determined at one wavelength, it can easily be determined for any other wavelength by use of the difference relation

$$n'(\lambda_1) - n'(\lambda_2) = \frac{\Delta m}{2t \cos I'} (\lambda_1 - \lambda_2)$$

The position of normal incidence ($\cos I' = 1$) could be found by rotating the plate until a null occurs (i.e., when the fringe pattern starts to expand for either direction of rotation). At this point the other parameters could easily be determined, except for $n'(\lambda_1)$.

The errors can be determined by the differential method:

$$dn'(\lambda_2) = dn'(\lambda_1) - \frac{(\lambda_1 - \lambda_2)}{2t \cos I'} d(\Delta m)$$

$$\begin{aligned}
& - \frac{\Delta m}{2t \cos I'} d(\lambda_1 - \lambda_2) + \frac{\Delta m(\lambda_1 - \lambda_2)}{2t \cos I'} \frac{dt}{t} \\
& + \frac{\Delta m(\lambda_1 - \lambda_2)}{2t \cos I'} \tan I' dI
\end{aligned}$$

To determine the total error, the following data will be used: $\lambda_1 - \lambda_2 = 1/2 \mu$, $t = 5 \text{ mm}$, and $\Delta m = 2$. It is assumed that one knows $n'(\lambda_1)$ to ± 0.00001 and that the beam impinges at normal incidence ($\cos I' = 1$); then for $\delta_m = 5 \times 10^{-3}$, $t = 2 \times 10^{-2} \text{ m}$, $I = 1.2 \times 10^{-7} \text{ rad}$, the allowable uncertainties are $d\delta_m = 0.01 \text{ fringe}$, $\delta\Delta\lambda = 10^{-2} \mu$, and $dt = 0.01 \text{ mm}$.

This method (relative index measurement) seems to be the best of those considered, but depends on an initial determination of the refractive index. The physical parameters are well within experimental procedures. The variation of the index could easily be checked for the wavelength region, one fringe at a time, and a complete set of data could be obtained. The temperature variation could also be found if the sample were heated while the fringe pattern was observed. This method would also offer a check on the variation of the index measured by another method.

From the experimental standpoint, two other problems are introduced: alignment of the instrument, and heat transfer in the sample. There are straightforward procedures available for the alignment of interferometers, but these are delicate and time-consuming.

If the absolute index can be determined by some other method, the interferometric technique is very good for determining any variation in the index as a function of wavelength and/or temperature.

3.4. SPHERE METHOD

This method of measurement utilizes multiple internal reflections through a solid Ge sphere.

A light ray incident upon a homogeneous, isotropic, transparent sphere with an index n is refracted into the sphere and undergoes $p - 1$ internal reflections with a portion of the ray being refracted outside at each internal reflection. The total deviation of the ray from its initial direction, Φ , is given by

$$\Phi = 2I - 2pI' + (p - 1)\pi$$

where I is the angle of incidence and I' is the angle to the normal after refraction (obtained from Snell's law). A particular ray which satisfies the equation

$$\sin^2 I = \frac{p^2 - n^2}{p^2 - 1}$$

is called a "limiting ray." This equation can be satisfied if $p \geq 2$ and $n < p$. It can be shown that ϕ is a minimum for a limiting ray; when the emerging light is viewed, therefore, a concentration of light is observed close to the limiting ray and, on one side of the bright band, there will be darkness. (The bright side is actually a diffraction pattern.)

Calculation also shows that the direction of the limiting ray depends on the refractive index of the sphere through the relation

$$\frac{d\phi}{dn} = \frac{2}{n} \sqrt{\frac{p^2 - n^2}{n^2 - 1}}$$

Thus the determination of ϕ allows calculation of the refractive index.

The advantage of this method is that the change in ϕ for a given change in n is much larger than is the change in the minimum deviation angle of a prism for the same change in n .

In order to design an experiment to measure n to within ± 0.00001 , the required angle-measuring accuracy and the size of the sphere required must be determined. A third consideration is the amount of energy arriving at the detector for a given amount incident upon the sphere.

A detailed examination of the experimental procedure is necessary to establish the accuracy requirement on angle measurements. The position of a limiting ray is determined by measuring the positions of two minima in the diffraction pattern associated with the limiting ray. The angular difference in seconds in the two minima is equal to a constant P multiplied by the difference in the ν values for the two minima given in Table IV of Walther's thesis [14]. P can be determined and used with one of the ν values and the corresponding minima position in order to calculate the position of the limiting ray. If the accuracy in measuring the position of the minima is a , then the accuracy of the position of the limiting ray is $\sqrt{3}a$.

It is not desirable to have to measure the position of three limiting rays; e.g., those corresponding to $p = 5, 6, 7$. From these data, $\phi_5 + \phi_6$ or $\phi_6 + \phi_7$ can be obtained and n calculated without knowing the direction of the incident ray. If each limiting ray is known to have an accuracy of $\sqrt{3}a$, then the sum of two limiting rays will be known to $\sqrt{6}a$. A calculation shows that $\Delta(\phi_5 + \phi_6)$ must be 4 arcsec for $\frac{\Delta n}{n} = \frac{2 \times 10^{-5}}{4}$. Therefore $a = 4\sqrt{6} \approx 1.6$ arcsec. Thus angle position measurements must be accurate to 1.6 arcsec. If the limiting ray position is accurate to $\sqrt{3}a$, then

$$P\Delta\nu = \Delta\phi (\sqrt{3} \ 1.6),$$

or

$$\Delta\nu = \frac{(1.7)(1.6)}{P} \approx \frac{1.7 \times 1.6}{300} \approx 0.01$$

Therefore

$$\Delta\phi = 0.01 \ P$$

or

$$\Delta n = \frac{0.01 \ P}{d\phi/dn}$$

Walther gives P as

$$P = \left(\frac{h}{48}\right)^{1/3} \left(\frac{\lambda}{r}\right)^{2/3} (2.06266 \times 10^5)$$

where

$$h = \frac{(p^2 - 1)^2}{p^2(n^2 - 1)} \sqrt{\frac{p^2 n^2}{n^2 - 1}}$$

and the numerical factor is the conversion from radians to arcseconds. If this is used without the numerical factor and substituting from above,

$$\frac{\Delta n}{n} = 0.0014 \left\{ \frac{(p^2 - 1)^2}{p^2(p^2 - n^2)} \right\}^{1/3} \left(\frac{\lambda}{r}\right)^{2/3}$$

However, for $\frac{\Delta n}{n} = \frac{2 \times 10^{-5}}{4}$, $r \geq 11.2$ cm.

Thus in order to measure n to an accuracy of two parts in the fifth decimal place requires a Ge sphere about 23 cm in diameter and the capability of measuring angles to 1.6 seconds of arc. This required diameter is impractical and inconsistent with $P \approx 300$ which assumes $r = 4$ cm.

If the accuracy requirement is relaxed to $\frac{\Delta n}{n} = 1 \times 10^{-5}$ and the measuring accuracy is retained at $\alpha = 1.6$ arcsec, then the radius required is 4 cm; this is practical and consistent with the value of P used. Note that it is not allowable to relax the measuring accuracy, as this will change the numerical factor and again an impractically large sphere will be necessary.

The third consideration is the required energy. The absorption coefficient of Ge is approximately 1 cm^{-1} at 15μ . The pathlength in the Ge is $\approx 2(p \times r)$. Therefore, for $p = 5$ and $r = 4$ cm, the attenuation factor is about $e^{-40} \approx 4 \times 10^{-18}$. Even if all other attenuation factors are neglected, the experiment is impossible because of internal absorption within the Ge. If the sphere size is reduced to the point where a signal is detectable, then the accuracy is less than 1 part in the fourth decimal place.

4 TRANSMISSION MEASUREMENTS

A sample of germanium which was submitted to the Weather Bureau by a vendor exhibited very poor transmission. The sample was investigated in order to learn why and to become further acquainted with the engineering problems. Transmissions of sections of this sample were measured and the mechanical yield properties and their relation to the transmission investigated.

4.1. STANDARD TRANSMISSION MEASUREMENTS

After the very low transmission of the 1-inch thick sample was measured, it was cut in half and the new surfaces were ground and polished. During the surface preparation, care was exercised to minimize strains and surface stresses introduced during the cutting and polishing operations. The final surface finish was microscopically comparable to that observed on the original (1 by 0.75 inch) faces of the sample.

These new sections were arbitrarily designated A and B, and the transmission characteristics of each were again determined. Marked differences were observed in the transmission peaks in the $1.8\text{-}\mu$ region. Sample A showed a peak transmission of about 13%, whereas the peak of sample B was 24%. These results indicate pronounced structural differences through the cross section of the original sample, and different numbers of free carriers.

These variations were studied further. Sample A and sample B were each cut in half, providing four sections 0.203 ± 0.001 inch thick after polishing. Figure 6 illustrates the transmissions of these sections, and relates each section to its position in the original sample. These results show that there is an increase in the free-carrier absorption from sample B to A; this indicates that this crystal was grown under nonuniform conditions. Sample B appears to contain fewer impurities than the other sections. Even here, however, grain boundaries were observed on the edges of the section and internal strains, lattice dislocations, and chemical

impurities could account for the decreasing absorption following the 2- μ peak. It is apparent, therefore, that this polycrystalline sample contains an impurity gradation from one side of the window to the other.

4.2. STRUCTURAL YIELD PROPERTIES

These properties were investigated by the dynamic spherical-indenter technique (described in Reference 15), which has been applied to both amorphous and crystalline structures; the characteristics of the defects produced by the indenter trace line are related to the internal energy and bonding characteristics of the material. The defects in glasses so produced are sensitive to variations in ionic substitutions within the network [16].

By examining the detailed variations in the flaw characteristics as a function of applied load, one can readily distinguish amorphous from crystalline structures, and determine the relative response of the given structure to applied stress. The mean length of the gross slip lines created by the indenter is designated as F_1 and is taken as a measure of the relative bond strength. The flaw number, designated F_n , is a function of the rigidity of the network. The product of these two parameters is an indication of the overall response of the given system to applied stress and is designated by

$$F_1 F_n = N_b \quad (1)$$

where N_b is a relative measure of the number of bonds disrupted. The relationship between stress and the radial distance from the point of loading on this spherical indenter is given by

$$\sigma = \left[\frac{\gamma - 2\nu}{2\pi} \right] P/r^2 \quad (2)$$

where P is the applied load and ν is Poisson's ratio. The critical stress σ_c occurs at the point where those flaws which were initiated at the trace line terminate; σ_c is found by substituting F_1 for r in Equation 2.

$$\sigma_c = kP/F_1^2 \quad (3)$$

where

$$k = (1 - 2\nu)/2\pi \quad (4)$$

If one assumes that σ_c is a constant for each type of structure and that crystal orientation is given, the flaw length is found to be related to the applied load by

$$F_1 = (k/\sigma_c)^{1/2} P^{1/2} \quad (5)$$

Equation 5 predicts a linear relation between F_1 and the square root of the applied load on the indenter. Thus under these given experimental conditions, the rate of flaw change with load is

$$\frac{dF_l}{dP} = \frac{1}{2} \left(\frac{k}{\sigma_c} \right)^{1/2} P^{-1/2} \quad (6)$$

From Equation 6, the energy of bond displacement expressed as flaw formation may be equated with the force exerted on the test surface:

$$\int dF_l = \left(\frac{k}{\sigma_c} \right)^{1/2} \frac{1}{2} \int P^{-1/2} dP \quad (7)$$

After integration, the functional relationship between flaw length and load is found to be

$$\bar{F}_l = \left(\frac{k}{\sigma_c} \right)^{1/2} P^{1/2} + K \quad (8)$$

where k is a constant and is physically related to the length of flaw created by a threshold stress which must be introduced into a given network to initiate the flaw formation. Equation 8 has been experimentally confirmed [2] and utilized in the analysis of germanium crystals. From Equation 8 and the empirical relationships, σ_c is given by

$$\sigma_c = k/a^2 \quad (9)$$

where a is the slope determined from the experimental curve.

The results of applying the indenter to two sections of the germanium sample are shown in Figure 7. Each point on these curves is the average of 60 measurements of the length of the gross slip or defect lines formed along the indenter traces on the polished germanium surface. These effects were displayed by etching the sample for two hours in a solution designated "number 2 etch" [17]. Although there is considerable scatter in the points (see Figure 7), the results generally follow the linear relationship given by Equation 8. The mean values for both the flaw lengths (F_l) and the flaw number (F_n) are given in Table II for sections B and C.

The critical stress can be calculated from the curves in Figure 7 by using Equation 9. These values are listed in Table III. The values of the calculated critical stress show a general agreement with the optical transmission characteristics shown in Figure 6. Section B, which shows the highest transmission, would also contain the lowest impurity level and fewer structural defects (e.g., slip lines and dislocations). Under an applied stress, section B would permit a greater degree of localized yield; a higher stress would therefore have to be applied before the

gross defect lines were formed. Section C, however, evinces a very high impurity level. This would make dislocation pinning more likely and would inhibit glide mechanisms; section C would therefore exhibit more brittle tendencies, and the σ_c value would be predicted to be lower.

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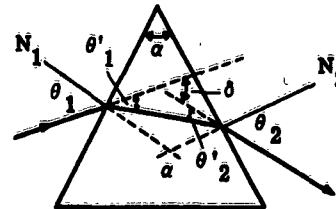


FIGURE 1. PRISM GEOMETRY FOR MINIMUM AND NORMAL DEVIATION

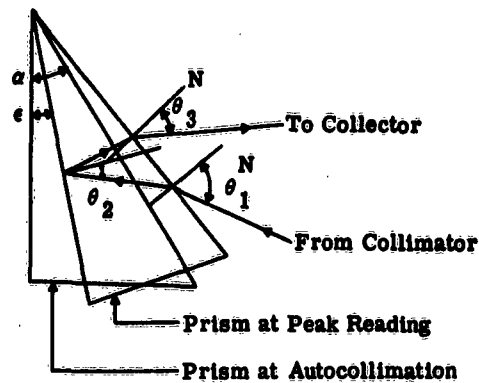


FIGURE 2. PRISM GEOMETRY FOR THE SCA METHOD

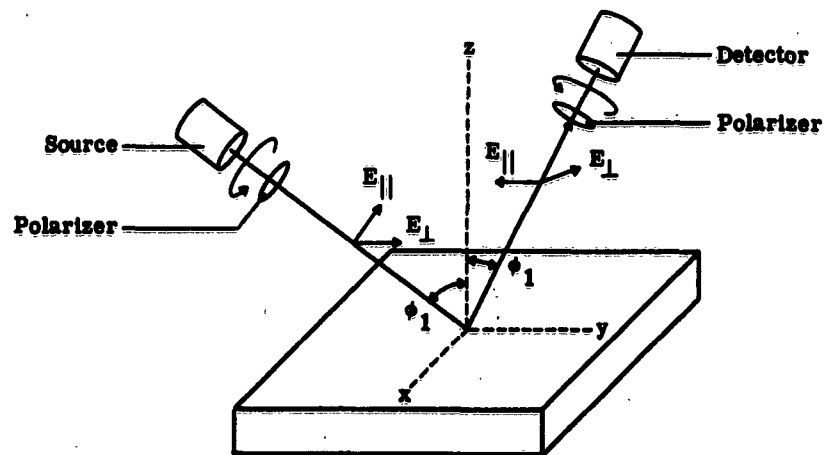


FIGURE 3. GEOMETRY FOR THE REFLECTION METHOD

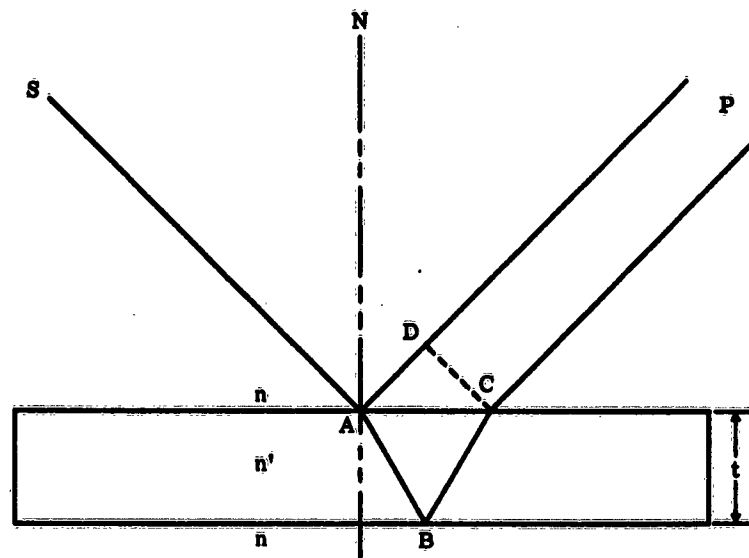


FIGURE 4. GEOMETRY OF INTERFERENCE AT A PLANE PARALLEL PLATE

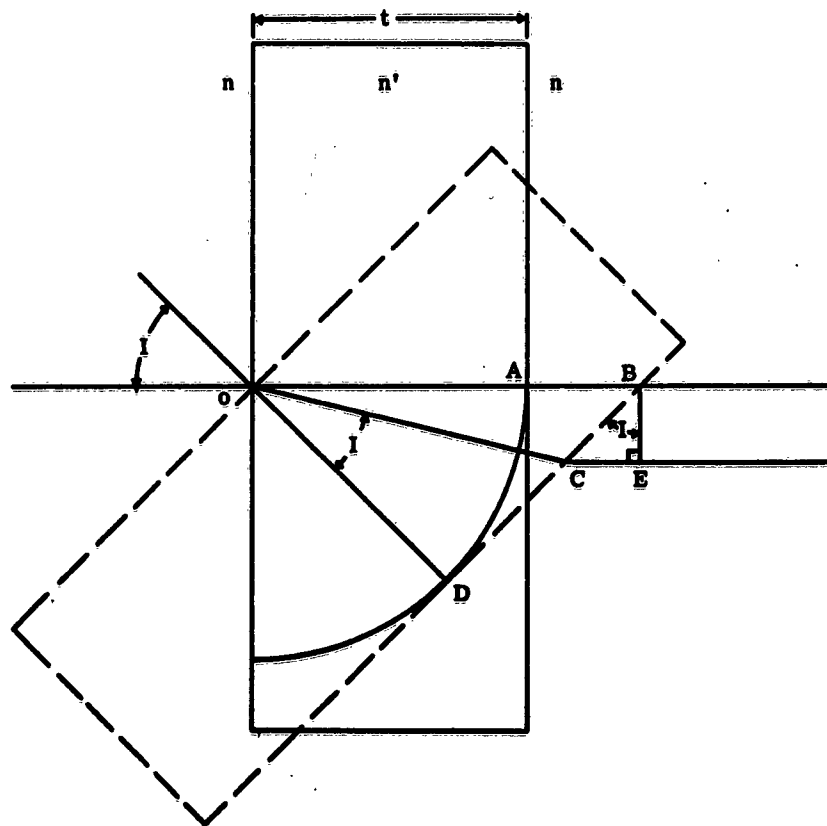


FIGURE 5. GEOMETRY FOR ROTATING PLATE METHOD

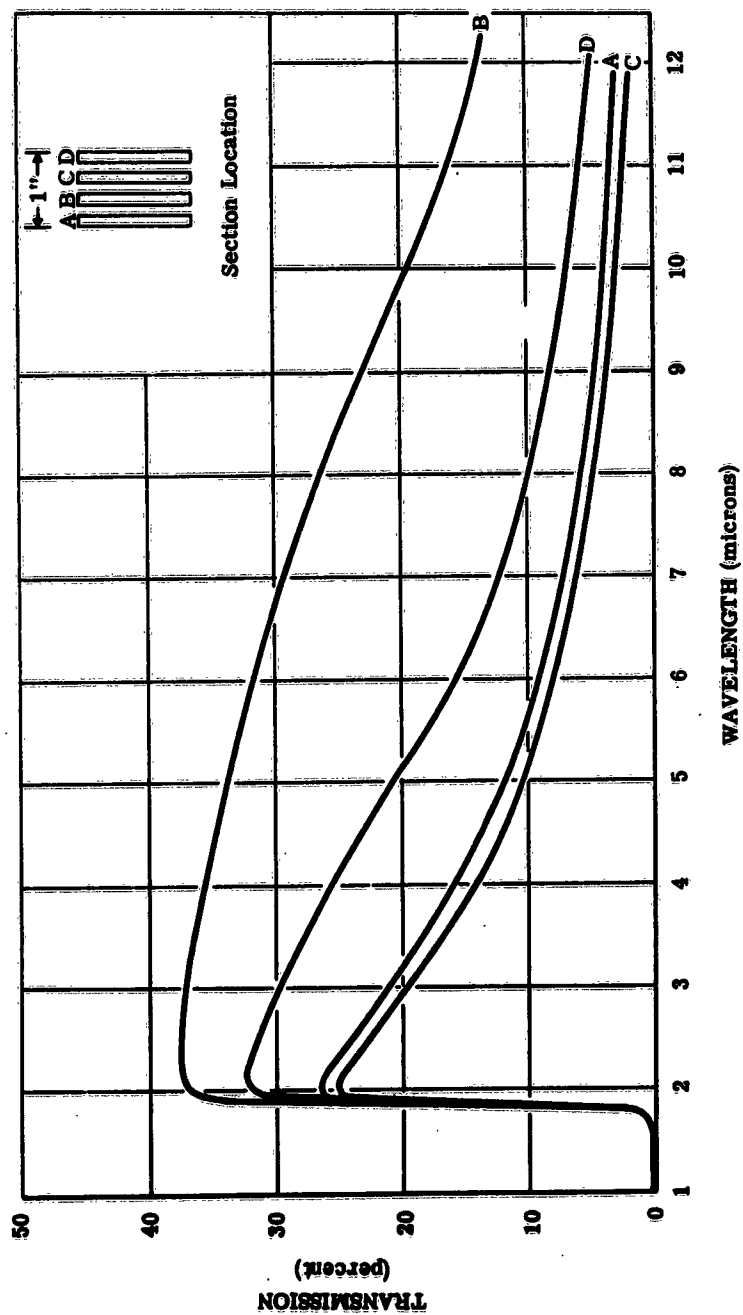


FIGURE 6. TRANSMISSION OF GERMANIUM SAMPLES

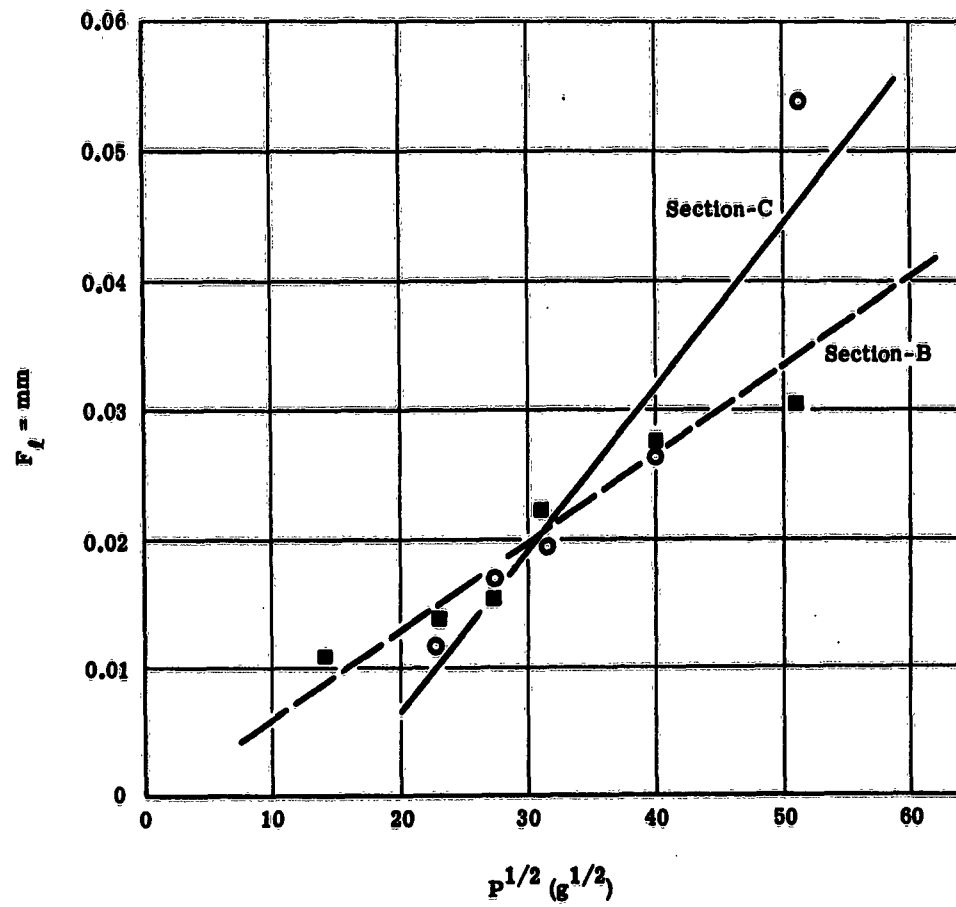


FIGURE 7. FLAW LENGTH AS A FUNCTION OF LOAD

TABLE I. ERRORS IN INDEX MEASUREMENT

	Minimum* Deviation	Two Position	Ditchburn	Tousey	Robinson	Interferometer	Rotating Plate Inter- ferometer	Sphere
δ (deviation angle)	2×10^{-3} mrad	7.89×10^{-3} mrad						
α (prism angle)	5×10^{-4} mrad	1.26×10^{-3} mrad						
λ (wavelength)	2.5 \AA at 3μ					10.5μ	$2 \times 10^{-6} \mu$	
n_{air} (index of air)	2.5×10^{-6}					10	5×10^{-7}	
T (temperature)	0.037°C			0.01				
β (rotation angle)		3.3×10^{-3} mrad						Errors are adjustable and interact; $\sim 10^{-4}$ is attainable.
ϕ (incidence angle)			0.05 mrad					
ψ (reflection phase)			0.05 mrad					
Δ (phase angle)			0.15 mrad					
θ_B (Brewster angle)					6.2×10^{-4} mrad			
I (incidence angle)						1 mrad	0.2 mrad	
t (thickness)						1.25 \AA	100 \AA	
m_0 (fringe order)						0.01	2×10^{-5} fringe	

*Errors are the same for simple deviation.

TABLE II. WAVELENGTH ERRORS

λ (μ)	$dn/d\lambda$ (μ) ⁻¹	$d\lambda$ (μ)
3	0.04	0.25×10^{-3}
10	0.0005	0.20×10^{-1}
13	0.000125	0.80×10^{-1}

TABLE III. CALCULATION OF CRITICAL STRESS
FOR GROSS DEFECT FORMATION

Section	σ_c (psi)
B	159,000
C	48,000

TABLE IV. INDENTER FLAW PARAMETERS IN SECTIONS FROM
A GERMANIUM CRYSTAL

Section B

P (grams)	F_1 (mm)	F_n (number per mm)	N_b
2600	0.0325	7.47	0.243
1600	0.0276	6.10	0.170
1000	0.0220	8.07	0.177
750	0.0155	7.14	0.111
500	0.0133	6.21	0.083
200	0.0111	4.43	0.049

Section C

P (grams)	F_1 (mm)	F_n (number per mm)	N_b
2600	0.0540	10.87	0.587
1600	0.0263	8.62	0.227
1000	0.0194	6.49	0.126
750	0.0169	6.02	0.102
500	0.0116	5.81	0.067